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Assessing the Effectiveness of Differentiated Instructional Approaches for Teaching Math to Preschoolers with Different Levels of Executive Functions

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Abstract: Previous studies have found that the development of mathematical abilities, along with the development of executive functions, predict students' subsequent academic performance. The present study aimed to assess the effectiveness of teaching the concept of area to preschool children with different levels of cognitive processes (CP) including executive functions and short-term memory. The experiment introduced the concept by using three different instructional approaches: traditional, contextual, and modeling. The sample included 100 children aged 6–7 years (M = 6.5 years), of whom 43% were boys. Each experimental condition included children with low, middle, and high levels of CP, as determined based on the NEPSY-II subtests. The children with low CP levels showed higher results in assimilating the notion of area after being taught using the contextual approach. In contrast, children with high CP levels showed a higher mastery of the concept of area following the use of the modeling approach. The results suggest the importance of CP development in building ways of mastering mathematical content. This contributes to choosing the optimal path of teaching mathematics for preschoolers, taking into account the development of their cognitive processes to improve their academic performance.

Keywords: executive functions; cognitive processes; teaching experiment; zone of proximal development (ZPD); mathematical skills

1. Introduction

1.1. Development of Cognitive Processes

The growth of executive functions is a key aspect of child development prior to school entry, which has been shown to predict preschoolers' future academic success [1–5]. In this study, we adhered to the concept developed by Miyake and colleagues [6], whose concept was modified by Adele Diamond, who understood executive functions as a group of cognitive processes that provide targeted problem solving and adaptive behavior in new situations [7]. They help to monitor, control thought, and activities by shifting the processes toward the task-related stimulus despite the presence of secondary tasks and interference [6,7]. Executive functions (EFs) are divided into the following main components: (1) working memory updating (visual and verbal); (2) cognitive flexibility (attention focusing and/or attention switching under conditions of changing targets), and (3) inhibitory control (an ability to inhibit a dominant response) [6–11]. These components are related to each other, but can also be considered as being independent of each other, which is why this model came to be called 'unity and diversity'.

Aside from EF, the development of short-term memory is important. Though there is debate in the scientific community about the differentiation or not between short-term memory and working memory, we have data today to accept that short-term memory is a simpler part of working memory [12]. There are two functions of memory [13,14]: (a) short-term memory as the ability to store material for short periods of time, and (b) the ability to store information while performing other cognitively demanding actions (updating working memory in EF). The situation where children are required to retain information in a nontransformed form may be related to individual differences in the ability to maintain information in an active, quickly retrievable state that subordinates controlling attention [15].

1.2. Relationship between Children Mathematical Skills and Cognitive Processes

There are significant changes in the acquisition of mathematical, spatial, and geometric abilities at preschool age [16–20]. Current research suggests that mathematical abilities, along with executive functions, predict students' subsequent academic performance [19], while the early differences in mathematical ability tend to persist with age [20].

The relationship between cognitive processes and mathematical ability has been confirmed by several studies indicating that working memory is directly related to children's ability to score success in preschool [20] and early school age [21]. Additionally, inhibitory control in preschool children [12] and primary school children [17] are predictors of mathematical ability at older ages. The development of cognitive flexibility in preschool children has also been found to allow for the prediction of mathematical success in children aged 7 years and older [17,22]. On the other hand, low levels of executive functions are associated with difficulty in learning mathematical skills and concepts [22,23]. Many current studies have also reported a direct link between preschool executive functions and the development of mathematical skills throughout schooling [16,17,22,24].

Some studies have found that visual–spatial short-term memory span is a predictor, specifically of math ability, while executive function skills predicted learning in general rather than learning in one specific domain [15]. In preschool, there is relation between verbal abilities and listening comprehension. Analyzing the relationship between text or listening comprehension and memory skills in preschoolers, it has been shown that short-term memory was predicted in the listening comprehension tasks [25].

These findings raise a fundamental question about the possibility of educating children with a low level of cognitive processes. Following Bruner [26], we believe that even in the case of children with a relatively low cognitive processes (CP) level, there is a fairly effective educational technology for mastering mathematical knowledge. In other words, we maintain that it is possible to build a zone of proximal development (ZPD) in the field of mathematical content for these children. According to Vygotsky, the ZPD is determined by the content of the tasks a child is yet unable to solve on his or her own, but is capable of solving in co-operation with other people, which gradually become internal mental processes: "The zone of proximal development defines the functions which have not ripened yet, but are in the process of ripening and which are due to ripen tomorrow but are in the embryonic state today" [27]. What interests us most is the question of finding adequate methods of mastering mathematical relations by preschoolers with different levels of cognitive processes.

1.3. The Aim of the Present Study

In order to compare the effectiveness of different instructional approaches for teaching mathematical skills to children with different levels of cognitive processes, it is necessary to choose mathematical content that is completely unknown to children. For this purpose, we chose the concept of area, which is typically not taught to children before elementary school. According to the Russian Federal Educational

Standards [28], the educational system begins to form this concept in second-grade primary school students (aged 8–9 years) by representing it as a property of flat geometrical figures. Thus, the mathematical concept chosen for the formative experiment was unfamiliar to our study participants because they did not receive any relevant instruction within their preschool educational program.

Three approaches were used to form the concept of area in preschool children, each of which emphasized one of the aspects of the formation procedure and defined the original ZPD construction.

According to the traditional approach adopted in the Federal Educational Standards [28], the notion of area is introduced using units of measurement. In this case, the ZPD was built based on a unit of area measurement within which communication between the teacher and preschool children is organized.

As an alternative way of forming the notion of area, we used a contextual approach in which area was also defined by measurement. This is introduced in the context of a narrative and is emotional and symbolic in nature [29]. The contextual approach is based on symbolization as a special form of content representation through the external form of an image. The peculiarity of symbolic mediation consists in that it operates in a situation of uncertainty, in which the methods of achieving the goal, the goal itself, or conditions are unknown. Mathematical transformations can also be symbolized. Uncertainty about mathematical problems traditionally creates difficulties for primary school students. The difficulty that arises in math lessons is related to the fact that the educational situation itself causes difficulties to children due to their uncertainty. The use of symbolic representation in teaching new concepts including mathematical ones will simplify this task. Thus, this approach is the "simplest" for children to learn, since it is a special form of activity, an intermediate between the game and educational [30].

The third is the modeling approach, in which the notion of area is introduced using spatial models that allow children to visualize the area in question and enable them to master the notion [30,31]. Most authors define the model as a means necessary to reflect a part of reality in order to gain a deeper insight into it [32]. Model building is understood as building a sign system, which implies a deeper understanding of the simulated content than that offered by the traditional approach [33,34]. Research confirms that the use of modeling as a method of teaching leads to much greater learning efficiency because the use of visual models significantly develops cognitive skills [31]. The visual model sets the ZPD of preschool children as a special space in which essential relationships are represented in clear-cut patterns.

Our hypotheses were as follows:

- Children with a higher cognitive processes development level will perform better as a result of learning through the modeling approach; and
- Learning through the contextual approach will be most effective for children with a lower cognitive process development level.

2. Materials and Methods

2.1. Participants

The sample consisted of 100 children aged 6–7 years (M = 6.5 years, SD = 4 months) attending a Moscow kindergarten, of which 43% were boys and 57% were girls. The research was conducted in the academic year of 2019–2020. All participants were learning in the same educational complex and had a traditional educational paradigm. The study was approved by the Ethical Committee of the Department of Psychology, Lomonosov Moscow State University, in accordance with the Declaration of Helsinki.

2.2. Instruments

The measurement of the students' CP level at the time of their education in the senior kindergarten group used the NEPSY-II subtest [35] of visual (Memory for Designs) and auditory short-term memory (Sentence Repetition), inhibition and switching (Naming and Inhibition), and cognitive flexibility (The Dimensional Change Card Sort). This allowed for the measurement of different components of cognitive processes in the students [30]. The following methods were used.

The NEPSY-II Memory for Designs (MFD) subtest was used to assess short-term visual memory. This method records the following final scores: content points are awarded for remembering image details correctly (max 46 points); spatial points reflect how correctly the child remembers a configuration (max 24 points); and bonus points are awarded to the child for correct memorization and simultaneous consideration of both parameters (max 46 points). All three indicators are summed up in the final score (max 116 points).

To assess verbal short-term memory, the NEPSY-II Sentences Repetition (SR) subtest was used. This method consists of 17 sentences, which gradually become more difficult to remember due to their length and grammatical structure. The child gets two points for each sentence he or she repeats correctly; one point if he/she makes one or two mistakes while repeating it by skipping, replacing, or adding words, changing word order; and if the child makes three or more mistakes or fails to answer he/she is awarded no points. The task is terminated if a child receives no points four times in a row.

The NEPSY-II Inhibition subtest aims to assess the level of information processing speed and inhibition. This technique consists of two blocks: a series of white and black shapes (circles and squares) and a series of arrows with different directions (up and down). Two tasks were carried out with each series of pictures: the task of shape naming (in this case, a child simply had to name the shapes he/she saw at a rapid pace and the inhibition task. In it, the child was to do everything the other way around: for example, if he/she saw a square, he/she was to say "circle " and so forth. Each task recorded the number of mistakes the child made and corrected, or failed to correct, and the amount of time the child spent doing the job.

The Dimensional Change Card Sort test (DCCS) [36] was used to assess cognitive flexibility. This method consists of three card-sorting tasks. First, the child is to arrange the cards by color, shape, and then following a complex rule: if the card has a frame, then he/she is to sort it by color, and if there is no frame, he/she has to sort it by shape. For each correctly sorted card, the child is awarded one point; at the end, the number of points for each try is calculated (max 6, 6, and 12 points, respectively) and then the total score for all the tasks is calculated (max 24 points).

A diagnostic was conducted with the children on an individual basis. Following the CP assessments, children were divided into three subgroups (low, medium, high), according to the results of the cognitive processes assessment using cluster analysis (K-means clustering) in IBM SPSS Statistics 22.

2.3. *Procedure*

The study was conducted in several stages. First, we assessed children's cognitive processes by the NEPSY-II subtests. Following the CP assessments (by the NEPSY-II subtests), children were divided into three subgroups (low, medium, high), according to the results of the cognitive processes assessment using cluster analysis (K-means clustering) in IBM SPSS Statistics 22. At the same time of CPs diagnosis, children were given several tasks to determine their initial understanding of the area concept (pre-test (see Section 2.3.1)). Participants in each subgroup (low, medium, high CP levels) were randomly assigned to three experimental conditions. Children took part in seven training sessions in their groups. The time duration of each lesson was the same in all groups (15 min). Finally, children received a post-test (a week after the end of the last class) at follow-up after one month (see Section 2.3.3).

2.3.1. Pretest Math Assessment

The pretest math assessment was the preliminary measure of how well the kindergarten students already understood the concept of area. For this purpose, the children were asked to do nine tasks consisting of different images, each involving a comparison of two rows of related shapes. The tasks were developed ad-hoc for the present research. In eight out of nine tasks, the figures differed in shape, number and/or location, but were equal in area. During the tasks, the child was asked the question, "Do the cows have the same amount of grass or do some of them have more than the others?", the student's answer and comments, if any, were recorded in a special form (see Appendix A).

2.3.2. Training Sessions

At the formative stage, children from each group were offered seven classes that were conducted in mini-groups of three children, each having two classes a week.

In the traditional (tick-box) approach group, the teacher introduced the children to the notion of area through the property of an object to occupy a certain area on a flat surface. The children learned how to compare the area of objects by superimposing their images and explaining the differences in the space occupied. This was followed by a transition to conventional yardsticks, whose role was played by geometric figures of different sizes. Thus, an object occupying more space on the plane (the table) was measured using large squares, while a small object (a notebook) was measured using smaller measurements. This contributed to the development of the children's understanding that it was possible to compare only those objects that were measured using one (the same) measurement. After that, the children measured the same subject using different sized measurements, which helped to determine the dependence of the results on the size of the measurement. By way of illustrating a problem situation, the teacher explained to the children that people had agreed to use a common measuring system to keep all measurements the same. Finally, the concept of a square centimeter was introduced, which was shown and given to children as a conventional yardstick. Square centimeters were also used to make several measurements, for instance, the area of small geometric shapes.

In the contextual approach group, the children were told a story about a sorceress who sent rain to wash little men's drawings off the asphalt. The children were to help the little men to protect their drawings from the rain with umbrellas (pre-cut paper cups). After that, the children, with the teacher's help, discovered that the round umbrellas failed to protect the whole drawing and that a square umbrella protected the drawing best of all. The children were given differently-sized paper squares as new umbrellas. As they covered each drawing, they found out that the whole drawing remained somewhat uncovered again. In the process, it was decided to make the umbrellas similar, then using the same squares. The children covered the differently sized drawings to determine how many little men with umbrellas they would have to call to protect the drawing "from rain". Having found out the difference, the children and their teacher concluded that the number of square umbrellas depended on the size of a drawing. At the next stage, the children were asked to cover a picture with small and large umbrellas to conclude that the number of little men depended on the umbrella size: the bigger the umbrella, the fewer little men you needed to call to cover the picture. Then, the children began to measure the surrounding space with square umbrellas, which they had agreed to call square centimeters. Next, the children were shown a square centimeter and asked to associate its size with that of the square in a notebook, with which the children began to measure the area of the drawn figures.

In the group taught with the modeling approach, the children were introduced to the concept of scale in the first stage. They "built" a house on sheets with differently-sized squares in order to understand that the resulting house size varied, depending on the square size. Then, by increasing and decreasing the square size, the children would change the size of their houses. The next step was to plan a small-scale layout of the room in which the lessons were being conducted. Little cubes were used as pieces of furniture; on the bottom of one of them, there were instructions indicating where to look for a little star. Upon finding the little star on the plan, the children were to look for it in the real room. Next, the concept of area was introduced as the number of squares that an object occupied. The children were offered different material objects (notebooks, notepads, and erasers) to determine their area by circling the outlines of the object with a pencil and counting the number of squares the drawn object occupied. After that, the concept of a square centimeter was introduced to the students who were then asked to determine the area of an object in square centimeters based on the calculation of squares that the object occupied in the notebook. Previously, the squares were measured with a ruler to determine that the square size was 1 cm^2. Then, it was explained to the children that each square could designate both 1 cm^2 and 1 m^2 if all agreed. Furthermore, the children were asked to solve engineering problems: to build a hangar for an airplane of a certain size and to determine whether a boat and a car drawn on a grid paper sheet would fit into the garages depicted on grid paper with larger squares.

Thus, all three approaches differed from each other in the way they introduced the concept of area, which was new to the students. In the traditional approach, the concept itself was given to children immediately, and then, with the help of conventional yardsticks, children learned to measure the area. The contextual approach implied the presence of an emotionally colored problem situation, in the solution of which children first learned to work with a conventional yardstick and indirectly came to the notion of area. In the modeling approach, the notion of area was not central either. Children came to this concept with the help of scaling and learned to use it by solving engineering problems and by comparing the sizes of objects measured in different scales.

2.3.3. Post-Test Assessment

The final measure of concept acquisition was made in the post-test and follow-up. The post-test was conducted one week after the end of the classes. Follow-up was carried out one month after the classes. In the first part of the final tests, the children were offered the same nine tasks as those in the preliminary tests to make final measurements of the development level of the students' ideas about area. At the follow-up, another series of four tasks were added, aimed both at measuring the level of the area notion and at reaching the ZPD by the students. For this purpose, they were asked to perform tasks in which 3D objects were used. For the students to solve them, it was necessary to take into account not only the area of the objects, but also their volume (depth/number, see Appendix B).

3. Results

The final sample size resulted in 82 observations that were used for analysis (Figure 1 shows the attrition analysis).

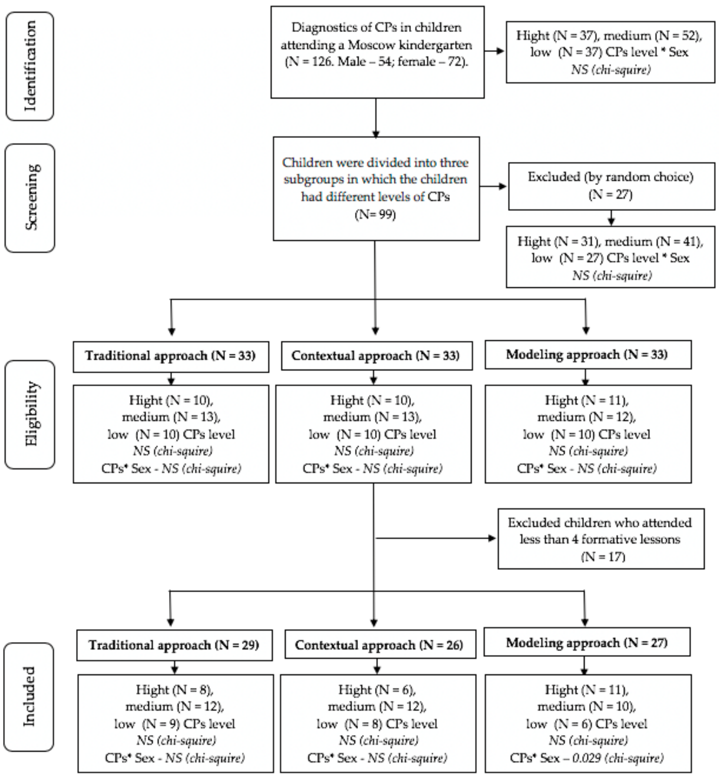

Figure 1. Attrition analysis for the sample size in this study. NS—not significant; *—cross-tabulation.

The groups of children included in the analysis did not differ from those who were excluded (Chi-square) for the first excluded group with 1.097 (p = 0.578) and 1844 (p = 0.103), correspondently). No significant statistical difference was observed in the subgroups included in the study, on the exception for differences in gender among children with low levels of cognitive development in the group of the modeling approach, where 83% were girls (Figure 1).

3.1. Descriptive Statistics

The distribution in the scales was mixed (Z ∈ [1.3–1.5]; $p \in [<0.001–0.155]$). The skewness values ranged from −1.259 to 0.210, and the kurtosis from −1.425 to 0.517. Thus, we used nonparametric methods for analysis because they are independent of the distribution.

The key goals of the analysis were to (a) examine whether/how different types of training influenced children's mastery of the concept of area and (b) determine whether/how these training effects were moderated by the initial level of the child's CP. Toward these goals, we conducted a cluster analysis (K-means method) of CP assessment results. All groups had significant differences (Kruskall–Wallis test, $p \leq 0.05$) between all CP components except for verbal short-term memory, naming errors, and inhibition corrected errors (Table 1).

Table 1. Results of the cluster analysis of students according to the cognitive processes level.

	Low CP Level (*n* = 37)	**Medium CP Level (*n* = 52)**	**High CP Level (*n* = 37)**	**The Significance of Differences**
	M ± SD	M ± SD	M ± SD	H-criterion *p*-value
Verbal STM (SR)	17.95 ± 4.5	19.00 ± 4.6	19.58 ± 4.6	H = 2.933; $p = 0.231$
Visual STM (MD Total)	62 ± 13.7	75.71 ± 19.4	82 ± 18.6	H = 21.83; $p < 0.001$
Cognitive flexibility (DCCS)	18.11 ± 2.2	18.9 ± 3.0	19.22 ± 2.8	H = 14.448; $p = 0.049$
Naming, uncorrected mistakes	0.86 ± 1.5	0.52 ± 1.0	0.5 ± 0.8	H = 0.469; $p = 0.791$
Naming, corrected mistakes	1.16 ± 1.2	0.85 ± 0.95	0.83 ± 0.73	H = 1.083; $p = 0.582$
Naming, time	54.89 ± 14.6	47.29 ± 10.8	46.35 ± 9.5	H = 9.23; $p = 0.01$
Inhibition, uncorrected mistakes	4.27 ± 7.05	4.88 ± 10.07	3.16 ± 6.9	H = 5.585; $p = 0.061$
Inhibition, corrected mistakes	1.84 ± 1.7	2.13 ± 1.8	1.86 ± 1.5	H = 0.533; $p = 0.766$
Inhibition, time	69.57 ± 17.5	61.62 ± 12.6	63.41 ± 17.2	H = 6.28; $p = 0.043$

The differences between groups were significant. However, their meaning (of both groups) were still within the same normative group range with regard to the age general norms [37].

Before conducting any further analysis, we will provide statistics on the distribution of students by class type in the formative experiment, depending on their level of CP (Table 2). In the course of the distribution, the students' initial level of CP (in the senior kindergarten group) was taken into account when forming the three classes. It should be noted that only children who attended more than four formative classes were selected for the final diagnosis and analysis.

Table 2. Analysis of the breakdown of students with different cognitive processes levels into groups of the formative experiment.

		Traditional Approach	Contextual Approach	Modeling Approach
High CP level	Frequency	8	6	11
	%	32%	24%	44%
Average CP level	Frequency	12	12	10
	%	35%	35%	29%
Low CP level	Frequency	9	8	6
	%	39%	34%	26%
Chi-square = 4.672; $p = 0.256$				

The table shows that statistically significant differences in the distribution based on the approach to teaching the concept of area to children were not found, so further analysis of group differences was justifiable.

All groups did not differ from each other in the pre-test (Table 3). Showing no differences in the pre-test is a requirement for understanding the intervention's results.

Table 3. Analysis of the breakdown of students with different CP levels into groups of the formative experiment.

	Traditional Approach	Contextual Approach	Modeling Approach	Kruskal–Wallis Criterion, Significance Level
	M ± SD	M ± SD	M ± SD	Chi-square = 2.852
Total pre-test score	4.36 ± 2.6	3.4 ± 2.6	3.7 ± 2.7	$p = 0.240$

3.2. Relation between Students' Cognitive Processes Scores and Pre- to Post-Test Changes in Their Math Scores

Comparison of the groups of children selected according to the CP level based on the results of the preliminary and two final tests (Table 4) also made it possible to verify significant differences between the groups (Mann–Whitney criterion, $p \leq 0.05$).

Table 4. Results of comparing the preliminary and final test results of children with different levels of CP.

	High CP Level (Mean Value, Standard Deviation)	Low CP Level (Mean Value, Standard Deviation)	Mann–Whitney Criterion, Significance Level
	M ± SD	M ± SD	
Total pre-test score	4.77 ± 2.4	3.15 ± 2.5	U = 779.0 $p = 0.008$
Score for the first part of the post-test	5.23 ± 2.4	3.7 ± 2.7	U = 206.5 $p = 0.061$
Total score for the post-test	7.31 ± 2.9	5.78 ± 3.3	U = 218.0 $p = 0.102$
Score for the first part of the delayed post-test	5.33 ± 2.3	4.08 ± 3.3	U = 245.5 $p = 0.088$
Total score for the delayed post-test	7.81 ± 2.7	6.28 ± 3.4	U = 241.5 $p = 0.077$

It should be noted that at the preliminary test stage, none of the students were able to cope with all nine tasks and more than thirty children coped with less than two tasks. After the formative classes, only seventeen students coped with less than two tasks, and five students succeeded in doing all the tasks.

As can be seen from Table 4, the groups of children with high and low CP levels differed significantly from each other, according to the results of preliminary tests. Differences between the groups of students in the first parts of the two final tests can be described as differences in the trend levels as well as differences in the overall test scores. However, according to the analysis of the mean values, the preschoolers with a high CP level were more successful in all tests.

For an in-depth analysis, we present a table of the mean values of the results in both final tests for preschoolers with different CP levels who were taught using different approaches.

Table 5 shows that students with a high level of CP performed better than those with medium and low CP levels as a result of learning the spatial concept via the modeling approach. However, the highest scores were obtained from the overall delayed test score for children with a medium level of CP who studied according to the traditional approach. For children with low levels of CP, the highest scores were achieved within the contextual approach. Thus, our first hypothesis may be considered partially confirmed. Moreover, the results show that it is possible to teach math to preschool children with different CP levels. In fact, each group of preschool children had their own ZPD created, based on the application of appropriate tools such as units of area measurement, a symbolic image, and a visual model.

Table 5. Mean values based on the post-test results for children with different cognitive processes levels who studied within the framework of different approaches.

	Study Approach	Score for the First Part of the First Final Test M ± SD	Score for the First Part of the Delayed Final Test M ± SD	Total Score of the First Final Test M ± SD	Total Score of the Delayed Final Test M ± SD
High CP level	Traditional	5.25 ± 1.75	5.75 ± 2.32	7 ± 1.93	8.13 ± 2.48
	Contextual	5.0 ± 3.08	4.4 ± 2.61	7.4 ± 3.36	7.0 ± 2.92
	Modeling	5.31 ± 2.72	5.43 ± 2.41	7.46 ± 3.43	7.93 ± 3.05
Average CP level	Traditional	5.7 ± 2.16	6.27 ± 2.24	7.2 ± 2.66	8.45 ± 2.98
	Contextual	3.92 ± 2.99	4 ± 3.05	6.08 ± 2.99	6 ± 3.36
	Modeling	3.75 ± 3.2	4.36 ± 2.62	6 ± 3.81	6.55 ± 2.98
Low CP level	Traditional	3.7 ± 2.93	3.6 ± 3.17	6 ± 3.64	6 ± 3.65
	Contextual	3.89 ± 2.93	5.5 ± 3.34	6.11 ± 3.26	7.88 ± 3.52
	Modeling	3 ± 2.55	3.14 ± 2.193	4.2 ± 3.271	4.86 ± 2.41

Note: The results that are maximal at every stage on the post-test for children with different CP levels, who studied within the framework of different approaches, are shadowed.

3.3. Analysis of the Effectiveness of Classes Types Depending on the Initial Level of CP

Friedman's F was carried out to test the second hypothesis according to which children with lower levels of CP development are likely to learn most effectively by using the contextual approach Table 6 shows the differential characteristics of variations between the two final test scores and similar pre-test tasks and shows the values of the F-test. The differences did not reach a significant level. However, according to the assessment of average values, it can be noted that the first follow-up tests resulted in the largest increase in the scores for students with low levels of CP as part of the traditional approach to learning. However, one month later, in the course of the delayed tests, children with low CP levels showed the highest increase in scores for all types of classes. Within the contextual approach, this increase was more than two points on average and was the highest relative to the other approaches to forming the notion of area, which speaks in favor of the proposed hypothesis.

Table 6. Differential characteristics of the differences in the two final test scores with similar pre-test tasks.

Teaching Approach	Differences in the First Final Test Scores with Similar Pre-Test Tasks			F Friedman Significance	Differences in the Delayed Final Test Scores of with Similar Pre-Test Tasks			F Friedman Significance
	High CP level	Medium CP level	Low CP level		High CP level	Medium CP level	Low CP level	
	M ± SD	M ± SD	M ± SD	F; *p*-Value	M ± SD	M ± SD	M ± SD	F; *p*-Value
Traditional approach	0.25 ± 2.05	0.7 ± 2.16	1.55 ± 2.96	F = 2.000 $p = 0.157$	0.75 ± 2.25	1.1 ± 2.7	1.5 ± 3.4	F = 1.190 $p = 0.275$
Contextual approach	0.2 ± 2.2	1.1 ± 2.79	0.01 ± 3.0	F = 0.800 $p = 0.371$	0.8 ± 2.04	0.83 ± 2.7	2.1 ± 3.09	F = 0.889 $p = 0.376$
Modeling approach	0.69 ± 2.8	− 0.17 ± 2.79	0.6 ± 3.57	F = 0.727 $p = 0.394$	0.92 ± 2.9	0.1 ± 2.02	1.15 ± 3.28	F = 2.130 $p = 0.144$
Total	0.46 ± 2.4	0.57 ± 2.6	0.73 ± 3.04	F = 3.276 $p = 0.071$	0.85 ± 5.53	0.72 ± 2.49	1.6 ± 3.16	F = 4.126 $p = 0.042$

Note: Differential characteristics of the differences that are maximal at every stage in the two final test scores with similar pre-test tasks for different approaches are shadowed.

To demonstrate the differences more clearly, observe the profile graphs for estimating the average values (Figure 2).

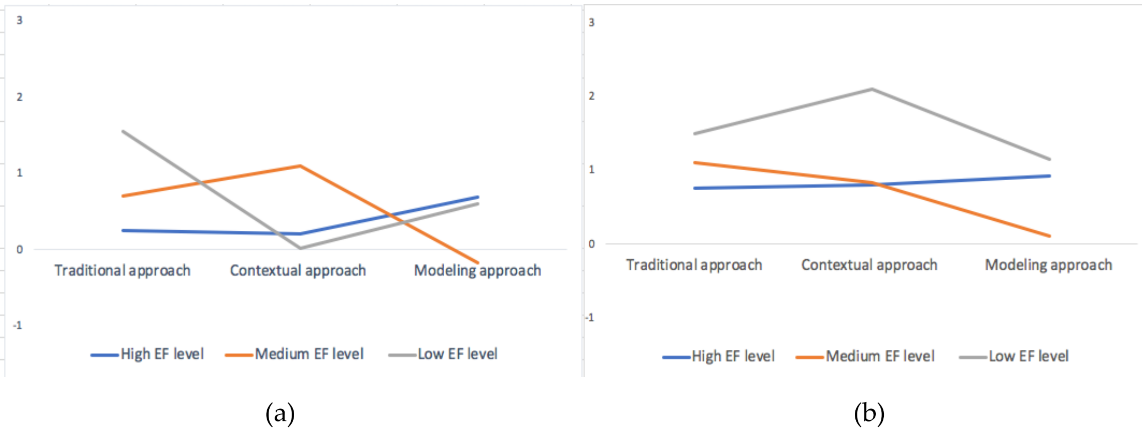

Figure 2. Comparison of average difference assessment between the first (**a**) and second (**b**) post-test results with similar pre-test tasks.

The assessment of the average difference between the pre-test tasks and similar tasks of the first final test (Figure 1) also shows that students with a low level of CP assimilated the tick-box approach significantly better (F = 5.329; $p = 0.007$) at this stage, gaining the greatest increase. In contrast, students with a high level of CP registered the greatest increase in the modeling approach. The contextual approach among children with both high and low CP showed the lowest growth rates.

However, students with low CP levels generally improved the most, particularly in the contextual approach where the most significant growth was recorded, as expected in our study. Students with high CP levels were almost equally successful in all three types of learning, while those with moderate CP levels were best served by the tick-box approach and were least successful when taught using the modeling approach.

We compared the overall results of both final tests with each other (Figure 3). We found that in the first nine tasks, the group of students with low CP levels who had studied using the contextual approach had the highest increase and students with low CP levels who had studied using the tick-box approached had the lowest increase. For students with high levels of CP, the contextual approach also recorded the greatest difference, and the modeling approach recorded the smallest difference. Students with average levels of CP scored the smallest score increase between the results of both tests within the contextual approach.

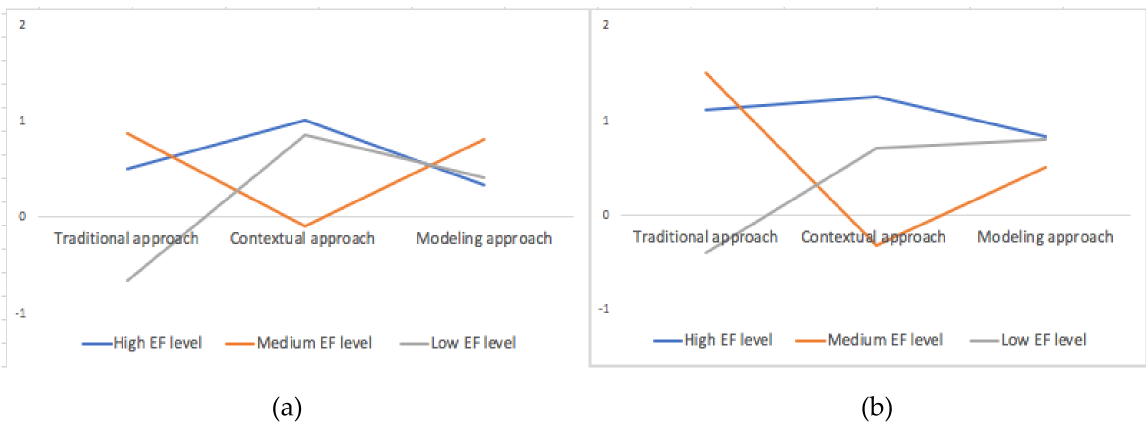

Figure 3. Comparison of the assessment of average differences between the first nine tasks (**a**) and of all tasks (**b**) of both post-tests.

The difference in the final scores of the delayed and subsequent tests on all of the above tasks shows that students with low CP-levels who learnt through the contextual and modeling approaches had the highest score gains. In contrast, students who initially had an average level of CP were still more likely to embrace the tick-box learning approach. In other words, the results indicate that children with different levels of CP development can learn successfully, but this requires an adequate educational method involving the use of appropriate means: units of area measurement, a symbolic image, and a visual model. In this way, each group of the preschool children had a ZPD of its own, which differed not only in the means used, but also in the nature of communication between teacher and children (i.e., the technology of construction).

4. Discussion

Based on the analysis of the results, it can be argued that the initial development level of the students' cognitive processes is directly related to children's ability to assimilate new mathematical concepts. This is consistent with the earlier findings that the EF level is a predictor of mathematical skill development [15,18,22,38].

The groups we identified by CP level were characterized by the most significant differences in inhibition, cognitive flexibility, and visual short-term memory, which as this study suggests, are the most closely related to the students' mathematical abilities [15–17,22].

In our preliminary tests, where we examined how fully the students formed the spatial notion that they had not previously studied, these groups also differed significantly from each other. Based on the results of the two subsequent tests, this difference remained throughout the experiment, which confirms the existence of a connection between the CPs and the development level of the students' mathematical abilities.

The fact that students with low CPs levels scored the highest post-study score increased generally and particularly in the contextual approach may indicate the importance of choosing an appropriate method of mathematical education given the level of CP formation for student development [39].

Additionally, cooperative problem solving with peers plays an important role in young children's cognitive growth. This allows children to create and sustain their own and joint goals, which likely influences their learning from group interactions [40].

Playing linear-number board games shows an effect on enhancing preschoolers' numerical knowledge and ability to acquire new numerical knowledge. Children who had played a linear number board game generated more correct answers and better quality errors in response to subsequent training on arithmetic problems; a task hypothesized to be influenced by knowledge of numerical magnitudes [41].

Our findings suggest that students with low CPs levels may exhibit the highest increase in knowledge while learning new mathematical concepts in groups (the concept of area being a case in point) through the contextual approach, where concepts are introduced in an emotionally constructed narrative context. Conversely, if the traditional (algorithmic) approach is applied to such children, their mathematical skills develop much more slowly.

The contextual approach is based on symbolic representation as a special form of the mental representation of an object. Children use symbolism in play as a means of expression [29]. Mathematical transformations can also be symbolized. Thus, one of the possibilities for improving the effectiveness of teaching mathematical problem-solving to children with low EF levels and encouraging the development of a cognitive interest in mathematics is related to the introduction of play elements in the lessons. Play positively influences the development of emotional–volitional and intellectual spheres of its participants [42–44] and can be considered as a technology of applying the emotional factor. It should be noted that play acts as children's ZPD in some studies [45–47].

Thus, our hypothesis was partially confirmed.

Additionally, the study showed that a child's mathematical intelligence is polyphonic, and is configured throughout the educational process by several factors, one of which is the CPs and the other is the way the educational process including the emotional component is organized. Moreover, the way of organizing education is determined by the nature of the cultural means used by the teacher and sets the ZPD for the children's acquisition of mathematical knowledge. Our research showed that the ZPD could be built in different ways, which may be effective to varying degrees depending on the CP development level.

Limitations: The research has several limitations. First, the group composition varied in the learning process, depending on which child was present at the time in the educational institution. Second, not all children completed the entire course, which made us include in our final calculations only those children who attended more than half of the classes. Thus, while the groups originally consisted of an equal number of students, they differed in number at the end of the formative experiment, which may also have affected the validity of the differences we obtained. Furthermore, all the participants were from one school.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Figure A1. An example of a task from the pre-test of the level in assimilating the notion of area. The question is "Do cows have the same grass, or does someone have more?".

Appendix B

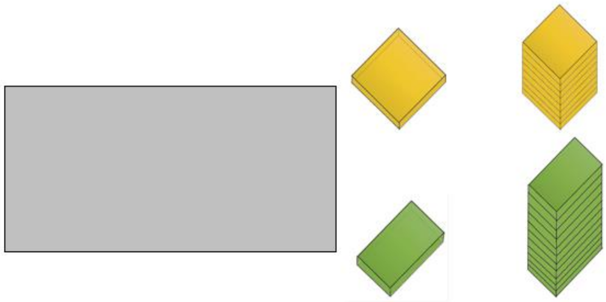

Figure A2. An example of an additional task from the post-test in assimilating the notion of area. The question is: "Which tiles are enough to cover the entire floor (gray rectangle): yellow or green?".

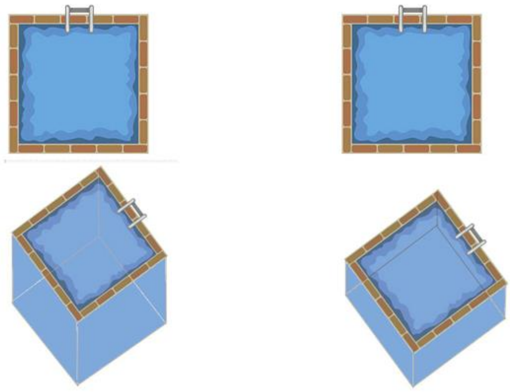

Figure A3. An example of an additional task from the post-test in assimilating the notion of area. The question is: "Which of these pools hold more water?".

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